scatter, with discrepancies of slightly greater than 10% being observed. This larger amount of scatter is due to the greater measurement difficulty in obtaining the slope of the shock from the experimental data.

Comparison of the results of these correlations and Ref. 1, using the specific flight conditions chosen by Berman, are shown in Fig. 2. Included for means of comparison is the accurate form of Berman's analysis, i.e.,

$$\left(\frac{r_S}{R_N}\right)^2 = 2\left(\frac{R_S}{R_N}\right)\left(\frac{X}{R_N} + \frac{\Delta}{R_N}\right) - e_s\left(\frac{X}{R_N} + \frac{\Delta}{R_N}\right)^2 \quad (2)$$

where

$$R_S/R_N = 2.09(1/k)^{0.1958} \qquad \Delta/R_N = 0.88(1/k)^{1.053}$$

with the values of the eccentricity factor e_s being expressed as functions of $1/k$ and specific locations of $(X/R_N + \Delta/R_N)$. In addition, Fig. 2 shows the results of Berman's approximate form, i.e., after approximating the eccentricity data with a single analytical expression and utilizing the forms for R_S/R_N and Δ/R_N as functions of density ratio, Eq. (2) was expressed as

$$\frac{r_s}{R_N} = \left\{4.18\left(\frac{1}{k}\right)^{0.1958}\left[\frac{X}{R_N} + 0.880\left(\frac{1}{k}\right)^{1.053}\right] - 0.646\left[\frac{X}{R_N} + 0.880\left(\frac{1}{k}\right)^{1.053}\right]^{1.467}\right\}^{0.5} \quad (3)$$

These expressions [Eqs. (2) and (3)] may be compared to the correlation obtained by Gregorek and the present writer, i.e.,

$$\frac{r_s}{d_N} = \left(1.52\,k^{-0.2} + \frac{1}{M_\infty^2}\right)\left(\frac{x}{d_N}\right)^{0.76k^{-2} + 1/(2M_\infty^2) + 0.44} \quad (4)$$

The results of these correlations, shown in Fig. 2, show relatively good agreement at Mach 5 and 17.6. However, the flight condition associated with the Mach 30 case indicated poor agreement because the limits of the empirical correlation of Ref. 2 had been exceeded, i.e., $k > 16$. Although the accuracy of Berman's results is unquestionable, the empirical correlation of Ref. 2 provides a comparatively simple expression extending over a wide range of density ratios, e.g., from 2 to 16, with an acceptable degree of accuracy. The advantage gained in using the results of Berman's approximate analysis [Eq. (3)] lies in the incorporation of the shock detachment distance in the over-all expression for the bow shock profile, whereas Eq. (4) does not contain this feature because of the choice of coordinate system. However, this easily may be rectified by use of Serbin's[8] or Ambrosio and Wortman's[9] results for values of the shock detachment distance.

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Comments on the Work Function of Metal Droplets

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THE papers presented by Rowe and Kerrebrock[1] and by Smith[2] have been of interest to the author in connection with our investigation of the work function of small metallic particles. Using classical image theory, Rowe and Kerrebrock[3] and Smith[2] derived two slightly different but completely equivalent expressions for the work required to remove an electron from a metal droplet. Their respective results are (the original notations and definitions are employed here)

$$W_Z = \varphi_s + \tfrac{3}{8}(e^2/4\pi\epsilon_0 A) + (Z - 1)e^2/4\pi\epsilon_0 A \quad (1)$$

and

$$W = e\varphi_w + \tfrac{3}{8}(e^2/4r_d) + (Ze^2/r_d) \quad (2)$$

in which W_Z is defined as the work required to remove the Zth electron from an initially neutral metal droplet and W is defined as the work required to remove an electron from a metal droplet of Z times charged. The flat surface work functions are φ_s and $e\varphi_w$, A and r_d are the droplet radii, and the rest of the notation is conventional. The purpose of this comment is to call attention to relevant earlier work, which can be profitably combined with either Eq. (1) or Eq. (2) to form a more complete expression for calculating the energy required to remove an electron from a metal droplet.

According to the elementary theory of metals, the electron work function of a metal φ is $\varphi = W - E_F$, in which W is the image energy and E_F is the Fermi energy. Assuming this equation also applies to finely dispersed metallic particles and denoting the bulk and dispersed states by the subscripts 0 and 1, respectively, we obtain the following expression for the droplet work function:

$$\varphi_1 = \varphi_0 + (W_1 - W_0) - (E_{F1} - E_{F0}) \quad (3)$$

A comparison of Eqs. (1) and (2) with Eq. (3) shows that, whereas the authors of Refs. 2 and 3 have accounted for the contribution of $\varphi_0 + (W_1 - W_0)$ to W_Z and W, they have not considered the possible contribution of the term $(E_{F1} + E_{F0})$. We shall see that this term is quite important, especially for neutral droplets with diameters less than about 100 Å.

In Ref. 4 it was pointed out that if a metal is dispersed, the kinetic energy of the electrons would increase, and thus would raise the Fermi energy of the metal. Using the simple model of an electron gas in a three-dimensional potential box for a metal, Zhukhovitskii and Andreev derived a first-order expression for the effect of dispersion on the Fermi

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energy of a metal. Their result is

$$E_{F1} - E_{F0} = (3.7/\Delta')(e^2/r_d) \quad (4)$$

in which Δ' is the ratio of the lattice period of the metal to the radius of the first Bohr orbit.

Equation (4) may be incorporated into either Eq. (1) or Eq. (2) to yield a more complete first-order expression for W_Z or W. Consider the combination of Eqs. (2) and (4); the resultant expression is

$$W = e\varphi_w + [\tfrac{3}{8} - (3.7/\Delta')]e^2/r_d + Ze^2/r_d \quad (5)$$

The importance of the quantity $(3.7/\Delta')$ can be assessed by computing its value for a particular metal. For example, the value of $3.7/\Delta'$ for potassium ($\Delta' = 8.0$) is 0.46, which is not only the same order of magnitude as $\frac{3}{8}$ but is actually larger and opposite in sign. Thus, for a neutral potassium droplet, the value of $(W - e\varphi_w)$ is $-0.09(e^2/r_d)$ instead of $+\frac{3}{8}(e^2/r_d)$.

Because of lack of direct data on the possible variation of droplet work function with droplet size, we are not able to confirm Eq. (5). However, there is evidence that Eq. (4) is quite reliable. This evidence is drawn from the theoretical work of Brager and Schuchowitzky[5] on the surface tension of molten metals.

It is well known that the surface tension of molten metals is much higher than that of the liquids. Brager and Schuchowitzky showed that such a high surface tension can be adequately explained on the basis that the dispersion of a metal results in an increase in the kinetic energy of the electrons in the metal. They were able to obtain reasonably good correlation between their computed values and the experimental values of surface tension for 12 (Na, K, Cu, Ag, Au, Zn, Cd, Hg, Sn, Pb, Sb, and Bi) of the 14 metals they considered.

The success of Brager and Schuchowitzky's theory, even though moderate, implies that Eq. (4) is probably quite correct. The basis for citing this implication may be found in Refs. 4 and 5. A close reading of these two papers will show that identical physical model and identical method were used to derive the basic equation for calculating the Fermi energy in Ref. 4 and the surface tension in Ref. 5. Because of this similarity in the basic approach to the theory of surface tension of molten metals and to the theory of Fermi energy of metal droplets, experimental confirmation of the surface tension theory in Ref. 5 may be taken to imply that the theory of Fermi energy of metal droplets as presented in Ref. 4, and consequently Eq. (4) of this comment, is basically correct.

It is interesting to note the variation of W with r_d as predicted by Eq. (5). By calculation one easily can show that the quantity $(\frac{3}{8} - 3.7/\Delta')$ is generally negative. Thus, for a neutral droplet the value of W is generally less than $e\varphi_w$ and varies inversely with r_d. Obviously, as r_d approaches the atomic radius, additional terms must be added to Eq. (5), so that W will approach the ionization energy I of a single atom. Since $I > e\varphi_w$, it is quite probable that the variation of W between I and $e\varphi_w$ will show a minimum at some value of r_d. Such a possible dependence of W on r_d is remarkably similar to the reported dependence of the photoelectric work function of thin films on film thickness.[6–8] Data in these references showed that, in thin films of aluminum,[6] silver,[6] gold,[7] and magnesium,[8] there was a characteristic thickness that yielded a minimum photoelectric work function. This minimum value was found to occur at 530 Å for Al, 80 Å for Ag, 52 Å for Au, and 230 Å for Mg.

In conclusion, it may be pointed out that in applying Eq. (5) to metal droplets, one should bear in mind that it is only a first-order equation, and as such it cannot be counted upon to hold for r_d smaller than a certain lower limit. However, within the framework of applying Eq. (5) as a first-order equation, we believe it is more complete than either Eqs. (1) or (2).

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Comments on "Physics of Meteor Entry"

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In her recent review paper on meteor physics, Romig writes (Ref. 1, p.392): "Studies in recent years, summarized in Ref. 21 (Ref. 3 of this Comment), show that the luminous efficiency factor can vary by more than two orders of magnitude. Some of this is due to differences in meteoroid composition and shape, but some is undoubtedly due to fragmentation and gas radiation." A similar statement is made also on p. 388: "The uncertainty in [the luminous efficiency factor] τ_0 (due primarily to composition and fragmentation) can be as large as two orders of magnitude"[16] (Ref. 16 is Ref. 4 of this Comment).

It must be pointed out that the uncertainty in the luminous efficiency is now much smaller, though it was indeed two orders of magnitude in 1959 and even later. After the experiments with artificial meteors[2] and my work[3] on Jacchia's precisely reduced photographic meteors, the uncertainty in τ_0 is of the order of two, as explicitly stated in Ref. 3. In conclusion, the meteor masses computed from the so-called luminosity equation have an uncertainty of a factor of two and not of two orders of magnitude; therefore, "the empirical approach of the Smithsonian group," to use Romig's words, is much more reliable and sound than it appears from Romig's conclusions.

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